Two dimensional inversion of magnetotelluric and radiomagnetotelluric data by using unstructured mesh

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SUMMARY

A new two-dimensional modeling algorithm for Magnetotelluric/Radiomagnetotelluric method is developed using unstructured mesh with finite element method**.** Modeling algorithm is used in developed inversion algorithm. The models can be represented by less number of elements and surface topography of the model can be designed more realistic with unstructured mesh than structured mesh. The algorithm is developed by using FORTRON90 programming language. Sparse matrix arithmetic is used to solve linear equation. The developed algorithm result is compare with analytical solutions and previously developed modeling algorithms. Jacobean matrix is calculated according to reciprocity. Smoothness Constrained Least-Squares method is used in inversion. In this study we only give preliminary inversion results of synthetic data.

Keywords: Finite Element, Unstructured mesh, Modeling, Inversion

INTRODUCTION

Radiomagnetotelluric (RMT) method is a newly developed geophysical method. There has been only few commercial application of this method. The RMT method has been used for shallow researches as engineering applications successfully (Turberg et al., 1996; Candansayar and Tezkan, 2006; Candansayar and Tezkan 2008; Bastani et al. 2012;).

We solve Helmholtz equations in forward modeling of MT/RMT methods. Rijo (1977) developed an algorithm which solves Helmholtz equation with FE method. Wannamaker et al. (1987), researched MT reflection of different topography models, Key and Weis (2006), Franke et al.(2007), added topography effect to forward modeling using unstructured triangle elements.

Regularized inversion methods use generally on inversion of MT/RMT methods (Rodi and Mackie, 2001, Candansayar, 2002,2008). Jacobean matrix calculation is most time-consuming part of inversion. We used reciprocity method to calculate Jacobean matrix (DeLugao and Wannamaker, 1996).

In this study, first, we compared 2D forward modeling algorithm result with analytic solution. Then preliminary inversion result of synthetic data will be presented.

2D MODELING ON RADIOMAGNETOTELLURIC AND RADIOMAGNETOTELLURIC

We solved 2D Helmholtz equations for MT/RMT methods. Finite element solution of Helmholtz equations for a triangular element is given as following for TM- and TE- mode respectively (Rijo 1987).

$$\left(\frac{-1}{4\sigma\Delta}\begin{pmatrix} b_i^2+c_i^2 & b_ib_j+c_ic_j & b_ib_k+c_ic_k \\ b_jb_i+c_jc_i & b_j^2+c_j^2 & b_jb_k+c_jc_k \\ b_kb_i+c_kc_i & b_kb_j+c_kc_j & b_k^2+c_k^2 \end{pmatrix} + \frac{i\mu w\Delta}{12}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}\right) f_n^e = S_e \quad (1)$$

$$\left(\frac{-1}{4iw\mu\Delta}\begin{pmatrix} b_i^2+c_i^2 & b_ib_j+c_ic_j & b_ib_k+c_ic_k \\ b_jb_i+c_jc_i & b_j^2+c_j^2 & b_jb_k+c_jc_k \\ b_kb_i+c_kc_i & b_kb_j+c_kc_j & b_k^2+c_k^2 \end{pmatrix} + \frac{\sigma\Delta}{12}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}\right) f_n^e = S_e \quad (2)$$

In equation (1) and (2), a, b and c is coefficients related with corner coordinates of triangle element, indexes i, j and k represents corner of triangles, Δ is area of triangle element. f_n represents electric field vector (E_y) for TE mode, magnetic field vector (H_y) for TM mode. S_e is a vector including known boundary values.

In the developed modeling algorithm we used Delaunay triangulation method. For this purpose, we use program named 'triangle' (Schewchuk 1996) to obtain unstructured mesh for forward solution algorithm.

Coefficients matrix, used to solve general matrix equation on MT/RMT method, is symmetric, and consists of a complex number. Linear matrix equation is solved by using PARDISO (Parallel Sparse Direct and Multi-Recursive Iterative Linear Solver) (Schenk ve Gärtner, 2006) solver. Matrices are stored in CSR (Compressed Sparse Row) format (Rose et al. 1972).

Solution of the developed MT/RMT modeling code is compared with analytic solution of fault model (Weaver et al. 1985,1986). Figure 1 shows fault model and comparison of analytical and numerical solution. Developed code is also compared with other forward modeling codes (Demirci and Candansayar 2010) for waste site model (Figure 2). In this model, we assume topography with 24 degree slope as a hill on waste area. Models generated with using structured and unstructured mesh is presented in Figure 2. Figure 3 shows TE- and TM-mode apparent resistivity and phase curves in RMT frequencies. It is seen clearly both solutions are in good agreement.

INVERSION OF RADIOMAGNETOTELLURIC AND MAGNETOTELLURIC DATA

According to regularization theory on inversion algorithms the following parametric functional is solved;

$$P(m,d)=\phi(m,d)+\alpha S(m) \tag{3}$$

where $\phi(m,d)$ is the misfit functional, α is regularized parameter, S(m) is the stabilizing functional. Candansayar (2002, 2008) compared inversion with various stabilizing functional. In this study we used smoothing stabilizing functional in regularization inversion. The solution of eq.(3) is given as following,

$$\begin{bmatrix} W_d A \\ \sqrt{\alpha} C \end{bmatrix} \Delta m = \begin{bmatrix} W_d \Delta d \\ -\sqrt{\alpha} C m_i \end{bmatrix} \tag{4}$$

where A is the sensitivity (Jacobian) matrix, Δd is the vector of differences between the measured and the calculated data, Δm is the parameter correction vector, C is the Laplacian operator, Wd is the data weighting matrix. We use reciprocity for calculation of jacobean matrix. (DeLugao and Wannamaker 1996; Candansayar, 2008).

DISCUSSION

In the developed algorithm, we used unstructured triangle elements. However, parameterization of each triangle is main issue in inversion part of the algorithm. We are discussing this issue in this presentation. The advantage of unstructured mesh is the surface topography can be represented better than structured grid with less number of element.

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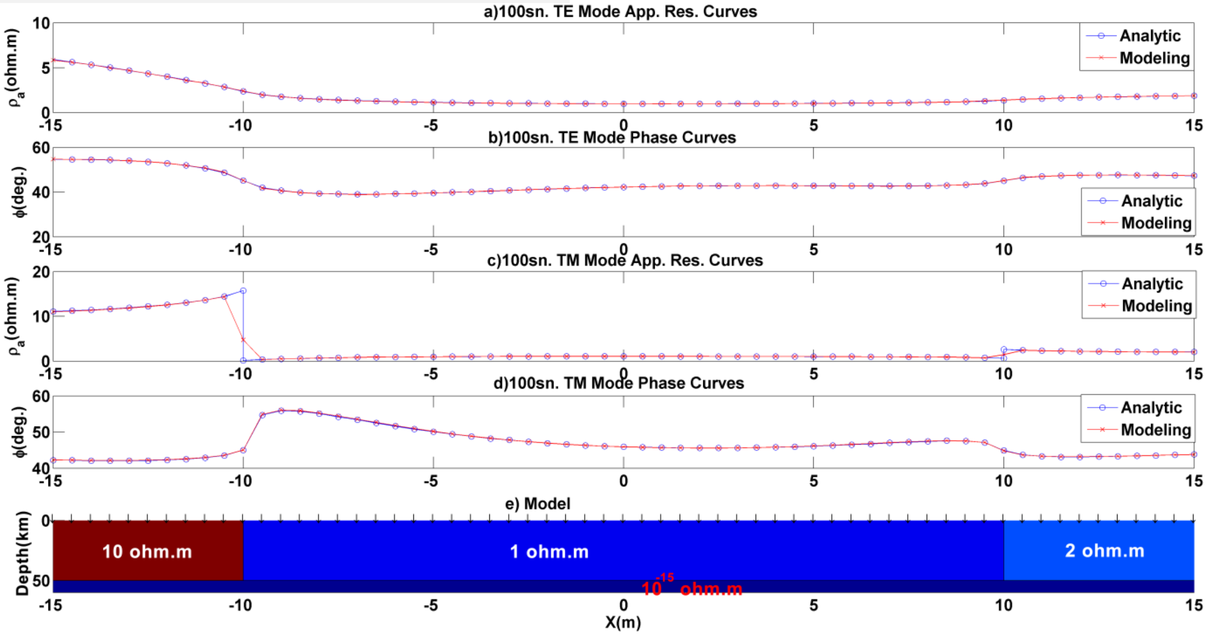

Figure 1. Fault model used for comparison between analytical solution and developed modeling code (Weaver et al., 1985)

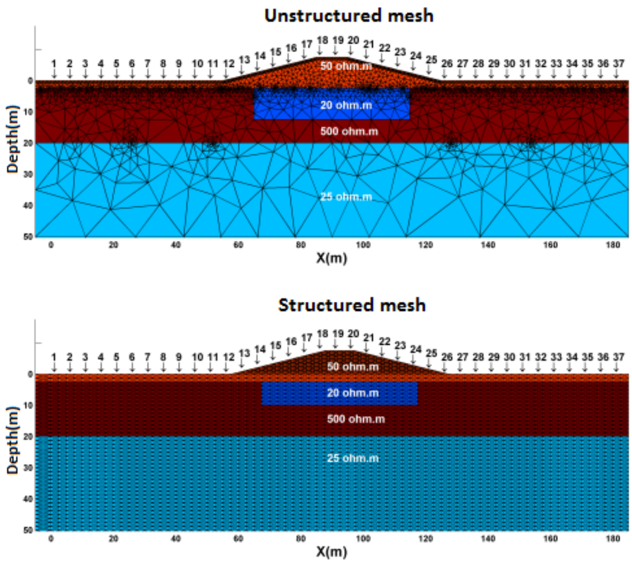

Figure 2. Models with topography using unstructured and structured meshes.

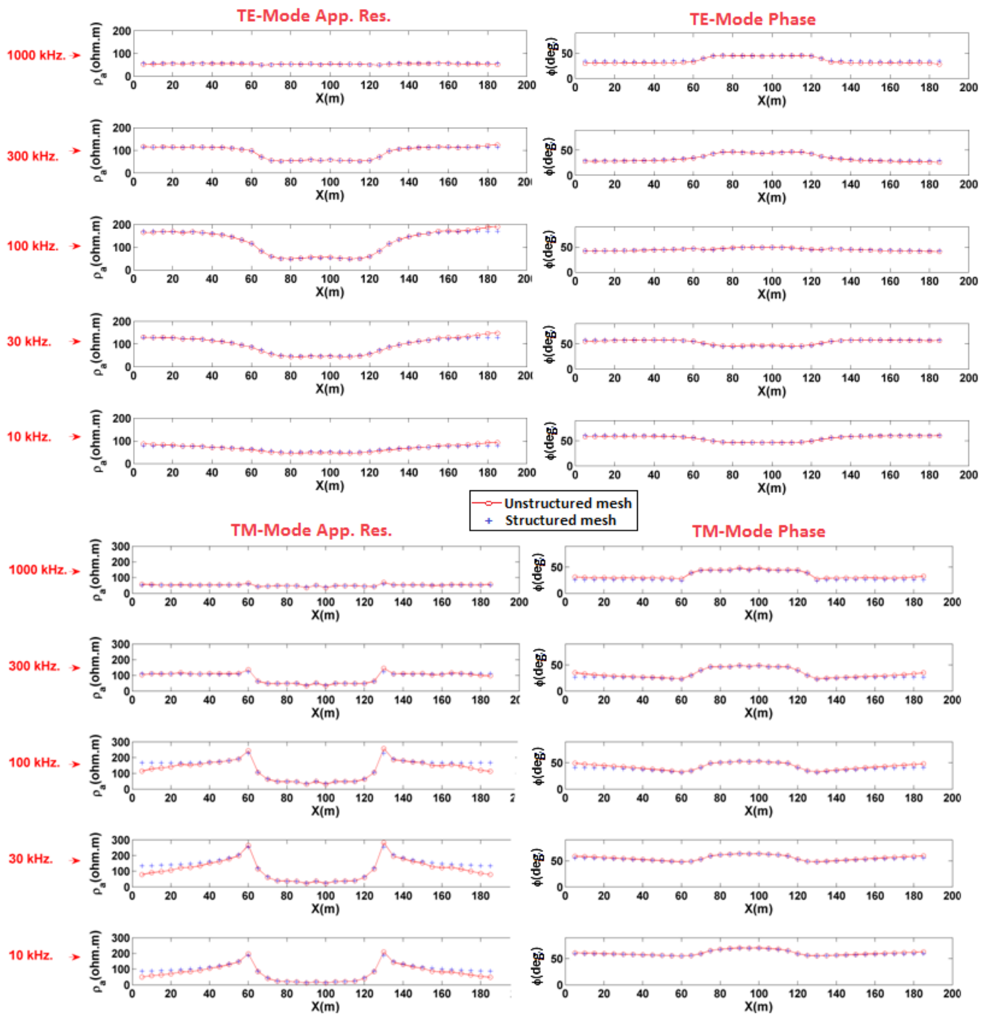

Figure 3. Comparison of modeling results using structured and unstructured meshes for TE- and TM-mode.