Joint Inversion of Magnetotelluric and Local Earthquake Data: Preliminary Results

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SUMMARY

Local Earthquake Tomography and Magnetotelluric methods are widely used in the investigation of deep subsurface structures. Tectonic interpretation is made based on subsurface velocity structure in seismology or subsurface conductivity structure in Magnetotelluric method. Recently, joint interpretation of geophysical data, sensitive to different physical parameters, has gained importance and joint inversion algorithms are developed. In this study, "Multi-stencil Fast Marching Method" including corner points in the discrete element model is used in the computation of the first travel times. Algorithm based on finite difference method using triangular element was used in the solution of the Helmholtz equation. A new joint inversion algorithm has been developed accounting these two techniques and by using Cross Gradient function. The results based on synthetic models show that the algorithm is more effective and the results are more informative according to the result obtained from a single method.

Keywords: Magnetotelluric, Local Earthquake Tomography, Joint Inversion, Cross Gradient Function, Multi-stencil Fast Marching Method

INTRODUCTION

In parallel with the development of computer technology, the usage of numerical methods in geosciences has become widespread. Accordingly, lokal earthquake tomography algorithms in order to determine underground velocity structure in Seismology (Aki et al. 1977; Rawlinson and Sambridge 2003) and Magnetoteluric (MT) inversion algorithms in order to determine underground resistivity structure in magnetotelluric method (Jupp and Vozoff 1977; Sasaki 1989; Uchida 1993; Rodi and Mackie 2001; Candansayar 2008) were developed. In the last decades with the increasing of computer processing capabilities, determination of subsurface structures by using multiple geophysical methods became applicable. More reliable information on geometric and physical properties of subsurface structures can also be obtained by simultaneous interpretation of different geophysical data. As a result, the usage of sequential (Vernante et al. 2002; Venisti at al. 2004; Wang et al. 2011) or joint (Gallardo and Meju 2003, 2007; Candansayar and Tezkan 2008) interpretation methods has gained importance.

In the following sections, Forward solution and joint inversion methods used in determination of deep subsurface structures were briefly explained and preliminary results of the developed joint inversion algorithm was discussed.

FORWARD SOLUTION

Forward solution in seismology

In seismology, measured quantity is the travel time of seismic waves propagating between source and receiver. The travel time *T* between a source (a) and a receiver (b) along a ray path for a continuous velocity field *v(x)* is given in integral form:

$$T = \int_a^b \frac{1}{v(x)} dl \tag{1}$$

where *v(x)* denotes the velocity at x coordinate in the model and *dl* is the integral unit along a ray path. Calculation of travel time between source and receiver is dependent on velocity distribution of the model. Therefore the following Eikonal equation must be solved:

$$\left(\nabla T\right)^2 = \frac{1}{v(x)^2} \tag{2}$$

Eikonal equation, Equation 2 was first solved by Vidale (1988) for two-dimensional space and in following decades has been studied by many researchers (Qin et al. 1992; Cao and Greenhalgh 1994; Sethian 1996; Rawlinson and Sambridge 2003; Hassouna and Farag 2007). All researchers solved Eikonal equation by using finite difference operators with different sort algorithms. In this study, we used Hassouna and Farag's (2007) approximation. In their approximation, the Eikonal equation was derived by using directional derivatives and then solved the equation using higher order finite difference schemes. In the solution of Eikonal equation, Finite Difference expression is given as:

$$\max(D_{ij}^{-x}T, -D_{ij}^{+x}T, 0)^2 + \max(D_{ij}^{-z}T, -D_{ij}^{+z}T, 0)^2 = \frac{1}{v_{ij}^2} \quad (3)$$

where v_{ij} denotes the velocity value at (i,j) point in the model, T is travel time, D^- and D^+ are backward and forward difference operators, respectively. In Equation 3, the diagonal elements are included similar to by Hassouna and Farag (2007).

Forward solution in magnetotelluric method

Frequency domain Maxwell equation is used to derive TE (Transverse Electric) and TM (Transverse Magnetic) mode Helmholtz equation for 2D forward solution of the magnetotelluric method. Helmholtz equation for TE and TM mode are given as:

$$(\nabla \times \nabla \times \vec{E})_y = \nabla^2 E_y = -i\omega\sigma\mu_0 E_y \quad (4)$$

$$(\nabla \times \rho\nabla \times \vec{H})_y = \nabla.\rho\nabla H_y = -i\omega\mu_0 H_y \quad (5)$$

where μ_0 is magnetic permeability of free space, E is electric field, H is magnetic field, ω is angular frequency and σ is conductivity. In the solution of the Helmholtz equation the most common numerical techniques are Finite Element (FE) and Finite Difference (FD) methods. Although the solution of equation is complex with using FE method, it has advantages in simulating complex resistivity models and incorporating topographic effect into the solution compared with FD method. However, solution and programming of the problem with FD method are easier. FD method also gives sensitive results same with FE method (Erdoğan et al. 2008; Demirci et al. 2012). Therefore, for forward modeling of the magnetotelluric algorithm, FD method was used to obtain apparent resistivity and impedance phase values. In this study, triangular cell definition were used not only improve the stability of the algorithm but also incorporate surface topography into the solution. For detailed information about two-dimensional forward solution and inversion algorithm can be referred to our previous studies (Candansayar 2008; Demirci 2009; Demirci and Candansayar 2010).

INVERSION

Joint inversion algorithms, sensitive to different physical parameters have been developed based on the two different bases. First; the different physical parameters are combined under a common parameter by using empirical relationships. However, the assumptions used in the determination of empirical relationships are useless for outlier cases. Second; the different physical parameters are combined under structural constraints and can be solved jointly. The mathematical basis of the method has been proposed by Haber and Oldenburg (1997).

Nowadays the most accepted approach is 'Cross Gradient Function' which is proposed by Gallardo and Meju (2003). In this method, as a result of the minimization of objective function, the parameter correction vector may be given as follows:

$$\Delta\mathbf{m} = N^{-1}n - N^{-1}B^T\left(BN^{-1}B^T + \alpha_B I\right)^{-1}\left[BN^{-1}n - B\Delta\mathbf{m}_{i-1} + t(\mathbf{m}_{i-1})\right] \quad (6)$$

Using matrix notation, the variables in Equation 6 are defined as follows:

$$\Delta\mathbf{m} = \begin{bmatrix}\Delta\mathbf{m}_R \\ \Delta\mathbf{m}_S\end{bmatrix},\quad n = \begin{bmatrix}\mathbf{A}_R^T\mathbf{W}_{dR}^T\mathbf{W}_{dR}\Delta d_R - \alpha_R\mathbf{C}_R^T\mathbf{C}_R\mathbf{m}_{i-1} \\ \mathbf{A}_S^T\mathbf{W}_{dS}^T\mathbf{W}_{dS}\Delta d_S - \alpha_S\mathbf{C}_S^T\mathbf{C}_S\mathbf{m}_{i-1}\end{bmatrix} \text{ and } \quad (7)$$
$$N = \begin{bmatrix}\mathbf{A}_R^T\mathbf{W}_{dR}^T\mathbf{W}_{dR}\mathbf{A}_R + \alpha_R\mathbf{C}_R^T\mathbf{C}_R & 0 \\ 0 & \mathbf{A}_S^T\mathbf{W}_{dS}^T\mathbf{W}_{dS}\mathbf{A}_S + \alpha_S\mathbf{C}_S^T\mathbf{C}_S\end{bmatrix}$$

where ***A*** is the sensitivity (Jacobean) matrix, ***Δd*** is the vector of differences between the measured and the calculated data, ***Δm*** is the parameter correction vector, ***C*** is the Laplacian operator, $\boldsymbol{W_d}$ is the data weighting matrix, ***t*** is cross gradients for all cells making up the model, ***B*** is the derivatives of ***t*** with respect to the model parameters. α_R, α_S and α_B are regularization parameter of resistivity, slowness and cross gradient, respectively. Subscript R and S describe resistivity and slowness, respectively.

In our inversion algorithm, a cooling approximation is used to find regularization parameters (α_R and α_S) (Candansayar 2008). In this approach, the square root of the maximum Eigenvalues of the each sensitivity matrix is used as initial α values and is halved at each iteration. In addition to previous studies, we have added a new regularization parameter "α_B" in to Equation 6. The main purpose of the new regularization parameter is to stabilize the algorithm and to control the contribution of cross gradient term in the solution.

MODEL STUDY

In order to test the capability of the developed algorithm, the resistivity and velocity models used to obtain synthetic data, were shown in Figure -1.a1 and b1. Especially complex models rather than simple ones were preferred and designed to force the algorithm stability. In the model (Figure 1.a1, b1), different resistivity and velocity contrasts and also one of the structural boundaries do not conform each other were used. According to separated inversion results, resistivity and velocity of the model and structural boundaries cannot be obtained correctly (Figure 1.a2, b2). On the other hand, when we look at the joint inversion results (Figure 1.a3, b3), it can be clearly seen that the harmony compared with the synthetic models are well (Fig. 1.a1, b1). Even in cases where the resistivity and the velocity boundaries of the parameters are different, the algorithm produces successful results.

CONCLUSION

The main highlights of this study can be given as follows:

- A joint inversion algorithm has been developed and tested on synthetic models.
- By adding the diagonal elements into the forward solution, more accurate ray paths and travel time is obtained via further minimization of the calculation error. The usage of Multi-Stencil Fast Marching algorithm in the joint inversion increases the stability and permits more accurate results.
- In addition to Gallardo and Meju's (2003) study, we added a new regularization parameter into the second term of parameter correction vector (Equation 6). The main purpose of the new regularization parameter is to stabilize the algorithm and control the contribution of cross gradient term to the solution. With this usage, we obtained more stable results.

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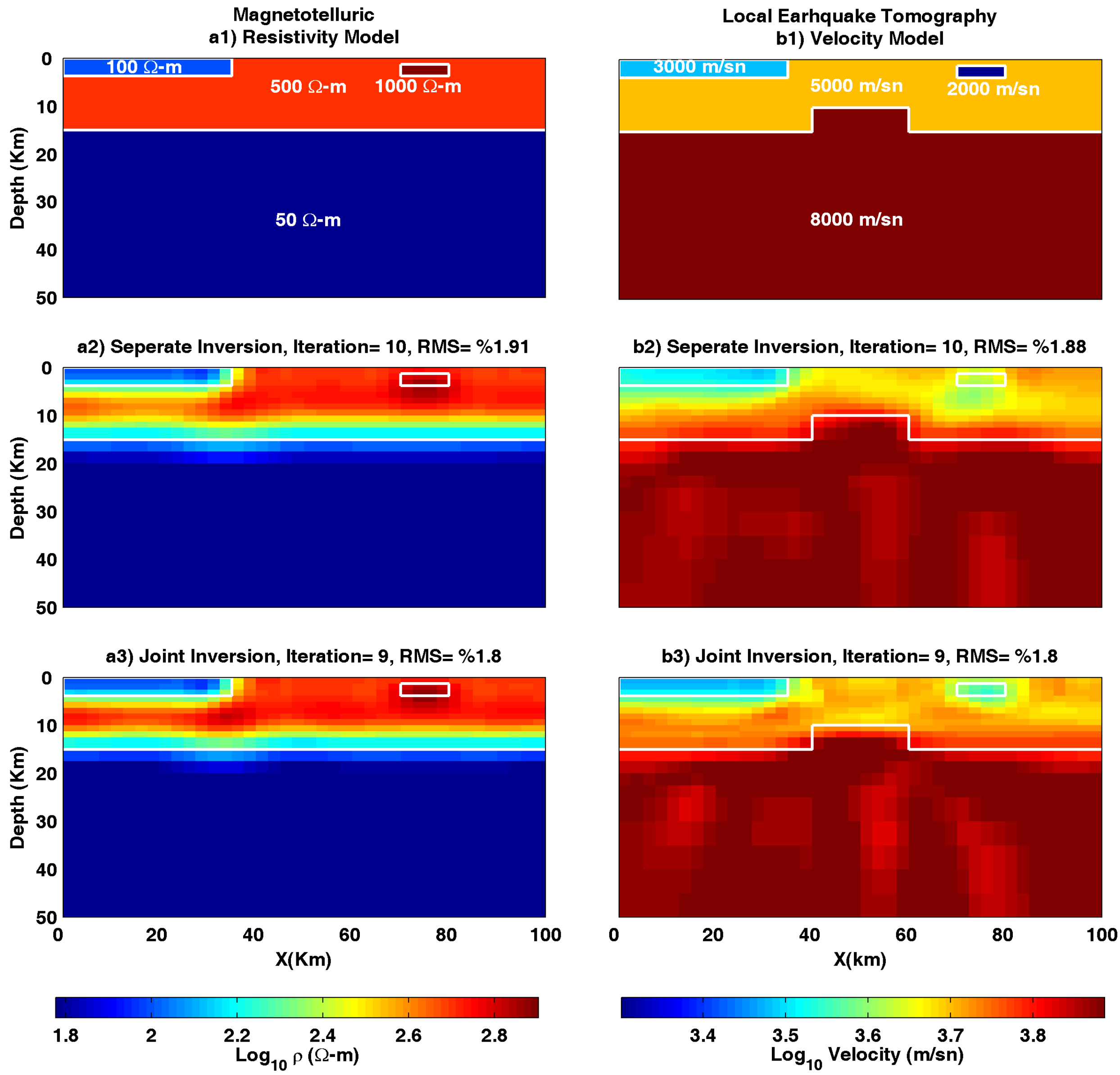

Figure 1. a1) Resistivity and b1) Velocity models used to produce synthetic data, a2) Resistivity and b2) Velocity model results obtained from separate inversion, a3) Resistivity and b3) Velocity model results obtained from joint inversion.